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Lagrangian Formulation for Free Mixed-Symmetry Bosonic Gauge Fields in $(A)dS_d$

K.B. Alkalaev, O.V. Shaynkman and M.A. Vasiliev

I.E.Tamm Theoretical Department, P.N.Lebedev Physical Institute,
Leninsky prospect 53, 119991, Moscow, Russia

alkalaev@lpi.ru shayn@lpi.ru vasiliev@lpi.ru

To the memory of Anatoly Pashnev

Abstract

Covariant Lagrangian formulation for free bosonic massless fields of arbitrary mixed-symmetry type in $(A)dS_d$ space-time is presented. The analysis is based on the frame-like formulation of higher-spin field dynamics [1] with higher-spin fields described as p-forms taking values in appropriate modules of the $(A)dS_d$ algebra. The problem of finding free field action is reduced to the analysis of an appropriate differential complex, with the derivation $\mathcal{Q}$ associated with the variation of the action. The constructed action exhibits additional gauge symmetries in the flat limit in agreement with the general structure of gauge symmetries for mixed-symmetry fields in Minkowski and $(A)dS_d$ spaces.

1 Introduction

The problem of finding manifestly covariant action for free higher-spin (HS) massless fields is to large extent the problem of identification of their gauge symmetries. The remarkable example is provided by Fang-Fronsdal theory of symmetric $4d$ HS gauge fields developed both on the flat [2] and AdS_4 [3] backgrounds that was later discussed and extended to any dimension within various approaches [4]-[17]. For mixed-symmetry gauge fields the problem becomes considerably more involved since, generically, the set of gauge symmetries required to describe an irreducible massless

field is different for Minkowski and AdS spacetimes[1] [18, 19]. Although there exist several formulations of the flat mixed-symmetry gauge field dynamics [21]-[28] the elaboration of a covariant formulation for generic massless fields in the $(A)dS_d$ was not known apart from numerous particular cases [1, 19, 29, 30, 31, 32, 33]. The light-cone formulation of generic massless fields in AdS_d was obtained, however, by Metsaev [18, 34].

By generalizing the frame-like formulation of symmetric fields developed in [10, 11, 12] it was proposed in Ref. [1] to describe a generic massless field propagating on the $(A)dS_d$ background as a p-form taking values in appropriate finite-dimensional module of the $(A)dS_d$ algebra ($o(d-1,2)$) $o(d,1)$. This scheme provides a natural realization of all gauge symmetries and trace conditions imposed on a mixed-symmetry gauge field in $(A)dS_d$. In addition, the method allows one to construct manifestly gauge invariant (linearized) HS curvatures. In this letter we announce construction of Lagrangian formulation for a bosonic gauge field of any spin (*i.e.*, symmetry type) in AdS_d within the approach of [1] [2].

The paper is organized as follows. In sect. 2 a short review of the frame-like formulation of mixed-symmetry fields is given. In subsect. 2.1 following [1], we introduce mixed-symmetry bosonic fields, describe their gauge symmetries and gauge invariant linearized curvatures. In subsect. 2.2 we reformulate all these objects in the manifestly antisymmetric basis for mixed-symmetry tensors most appropriate to the construction of the action functional. In sect. 3 we introduce a convenient oscillator approach. In sect. 4 a general form of manifestly covariant and gauge invariant HS action generalizing the MacDowell-Mansouri action for gravity [35] is formulated and the decoupling condition guaranteeing that the action describes the correct number of degrees of freedom is imposed. In sect. 5 and sect. 6 we reformulate the problem of finding free action in terms of the analysis of a certain differential complex. In sect. 7 we study the flat limit of the constructed action and show that it possesses all necessary additional flat gauge symmetries absent in the AdS space [18, 19]. Conclusions are given in sect. 8.

2 Frame-like formulation of mixed-symmetry fields

In the frame-like formulation of Einstein gravity with the cosmological term the gauge field 1-form[3] $\Omega^{AB} = -\Omega^{BA} = dx^{\underline{n}}\Omega_{\underline{n}}^{AB}$ is associated with basis elements of the AdS_d algebra. With the help of the compensator field $V^A(x)$ normalized

[1]Note that we restrict our consideration by elementary relativistic fields corresponding to the UIR's of the space-time symmetry algebra with the energy bounded from below. Beyond this class, the interesting case of the partially massless gauge fields is described in [20].

[2]Although bosonic fields in dS_d space can be described analogously to the AdS_d case, for definiteness we confine ourselves to the AdS_d case in this letter.

[3]We work within the mostly minus signature and use notations $\underline{m}, \underline{n} = 0 \div d-1$ for world indices, $a, b = 0 \div d-1$ for tangent Lorentz $o(d-1,1)$ vector indices and $A, B = 0 \div d$ for tangent AdS_d $o(d-1,2)$ vector indices. We also use condensed notations of [7] and denote a set of symmetric vector indices $(a_1 \ldots a_s)$ and a set of antisymmetric vector indices $[b_1 \ldots b_p]$ as $a(s)$ and $b[p]$, respectively.

as $V^A V_A = 1$ [36], Ω^{AB} can be covariantly decomposed into the frame field $\lambda E^A = \mathrm{d}V^A + \Omega^{AB} V_B$ and the spin connection $\omega^{AB} = \Omega^{AB} - \lambda\,(E^A\,V^B - E^B\,V^A)$. Parameter λ is the inverse radius of the AdS_d spacetime. (Note that λ has to be introduced to make the frame field E^A dimensionless.) The following relations hold $E^A\,V_A = 0$, $\mathcal{D}V^A \equiv \mathrm{d}V^A + \omega^{AB} V_B = 0$. In these notation, the background AdS_d geometry is described by the gauge field $\Omega_0^{AB} = (E_0^A,\, \omega_0^{AB})$, which satisfies the non-degeneracy condition $rank(E_{0\underline{n}}^A) = d$ and the zero-curvature equation

$$R^{AB}(\Omega_0) \equiv \mathrm{d}\Omega_0^{AB} + \Omega_0^A{}_C \wedge \Omega_0{}^{CB} = 0 \iff D_0^2 = 0\,, \qquad (2.1)$$

where $D_0 T^{AB\dots} \equiv \mathrm{d}T^{AB\dots} + \Omega_0{}^A{}_C \wedge T^{CB\dots} + \Omega_0{}^B{}_C \wedge T^{AC\dots} + \cdots$ (see, e.g., [1, 12] for more details). The AdS_d metric tensor is expressed in terms of the frame field E_0^A as $g_{\underline{mn}} = E_{0\underline{m}}{}^A E_{0\underline{n}}{}^B\,\eta_{AB}$.

2.1 Mixed-symmetry bosonic frame-type gauge fields

Let us consider a massless bosonic field $\Phi(x)$ corresponding to a spin $\mathbf{s}$ unitary module of Poincare ($iso(d-1,1)$) or AdS_d ($o(d-1,2)$) algebra in d dimensions. A spin $\mathbf{s}$ of a bosonic field is described by a set of integers $\mathbf{s} = (\underbrace{s,\dots,s}_{p}, s_{p+1},\dots,s_q)$, where $s > s_{p+1} \geq \dots \geq s_q$, $1 \leq p \leq q$ and $q = [(d-2)/2]$ for Minkowski case, or $q = [(d-1)/2]$ for the AdS_d case. $\Phi(x)$ can be described as a tensor field $\Phi^{a_1(s),\dots,a_q(s_q)}(x)$, which satisfies certain tracelessness conditions and Young symmetry conditions corresponding to the Lorentz Young tableau $Y(s,\dots,s_q)$ with q rows of the lengths $s,\dots,s_q$, *i.e.* it is symmetric in each group of indices $a_i(s_i)$ and its symmetrization over all indices from a given group $a_i(s_i)$ with an index a_j gives zero once $j > i$.

In Minkowski space, the physical degrees of freedom described by the field $\Phi(x)$ are characterized by a unitary finite-dimensional irrep $V(\mathbf{s})$ of the Wigner little algebra $o(d-2)$. Redundant degrees of freedom are removed by appropriate gauge symmetries [23, 24]

$$\delta\Phi^{a_1(s),\dots,a_i(s_i),\dots,a_q(s_q)} = \sum_{i=p}^{q} \mathcal{P}_\Phi(\partial^{a_i} S_i^{a_1(s),\dots,a_i(s_i-1),\dots,a_q(s_q)})\,, \qquad (2.2)$$

where gauge parameters $S_i^{a_1(s),\dots,a_i(s_i-1),\dots,a_q(s_q)}$ correspond to all allowed (*i.e.* nonzero) Young tableaux $Y(s,\dots,s_i-1,\dots,s_q)$ and $\mathcal{P}_\Phi$ is the projector on $Y(s,\dots,s_q)$.

In the AdS_d space, the physical degrees of freedom of the field $\Phi(x)$ are characterized by a unitary finite-dimensional irrep $V(E_0,\mathbf{s})$ of the maximal compact subalgebra $o(2) \oplus o(d-1) \subset o(d-1,2)$, where E_0 is the lowest energy eigenvalue. By definition, the lowest energy of a massless field $E_0 = s-p+d-2$ is such that the AdS_d representation $D(E_0,\mathbf{s})$ induced from $V(E_0,\mathbf{s})$ corresponds to the boundary of the unitarity region in the weight space [18]. As shown in [18, 19], in AdS_d the only gauge symmetry of the field $\Phi(x)$ is associated with S_p (*i.e.* with the first gauge parameter in (2.2))

$$\delta\Phi^{a_1(s),\dots,a_p(s),\dots,a_q(s_q)} = \mathcal{P}_\Phi(\mathcal{D}^{a_p} S_p^{a_1(s),\dots,a_p(s-1),\dots,a_q(s_q)})\,, \qquad (2.3)$$

while all other gauge symmetries are lost.

According to [1] the field $\Phi(x)$ can be equivalently described within the framework of the frame-like formalism analogous to the MacDowell-Mansouri approach to gravity [35], which was generalized to totally symmetric spin s gauge fields in [10, 11, 12]. In [1] it was proposed that dynamics of the metric-type field $\Phi(x)$ can be described by a p-form gauge field

$$\Omega_{(p)}{}^{A_0(s-1),\dots,A_p(s-1),A_{p+1}(s_{p+1}),\dots,A_q(s_q)}\,, \tag{2.4}$$

which takes values in the irreducible $o(d-1,2)$-module corresponding to the Young tableau $Y(\underbrace{s-1,\dots,s-1}_{p+1},s_{p+1},\dots,s_q)$ constructed by cutting the shortest column of the height p of the $o(d-1,1)$ Young tableau corresponding to the $\Phi(x)$ and then by adding the longest row of the length $s-1$. The linearized HS curvature $(p+1)$-form associated with the gauge p-form field (2.4) is

$$R_{(p+1)}{}^{A_0(s-1),\dots,A_q(s_q)} = D_0\Omega_{(p)}{}^{A_0(s-1),\dots,A_q(s_q)}\,. \tag{2.5}$$

The curvature $(p+1)$-form is manifestly invariant under the gauge transformations

$$\delta\Omega_{(p)}{}^{A_0(s-1),\dots,A_q(s_q)} = D_0\xi_{(p-1)}{}^{A_0(s-1),\dots,A_q(s_q)} \tag{2.6}$$

with the $(p-1)$-form gauge parameter $\xi_{(p-1)}{}^{A_0(s-1),\dots,A_q(s_q)}$ and satisfies the Bianchi identities

$$D_0R_{(p+1)}{}^{A_0(s-1),\dots,A_q(s_q)} = 0 \tag{2.7}$$

as a consequence of the zero-curvature equation (2.1).

Reduction of the p-form gauge field (2.4) with respect to the Lorentz subalgebra $o(d-1,1)\subset o(d-1,2)$ gives a set of p-form Lorentz-covariant gauge fields containing physical field, auxiliary fields and extra fields associated with different irreducible $o(d-1,1)$-modules explicitly described in [1]. In particular, the physical field, being analogous to the frame field in gravity, is

$$\lambda^{s-1}e_{(p)}{}^{A_1(s-1),\dots,A_q(s_q)} = V_{A_0}\cdots V_{A_0}\Omega_{(p)}{}^{A_0(s-1),A_1(s-1),\dots,A_q(s_q)}\,, \tag{2.8}$$

i.e. the physical field $e_{(p)}$ is the maximally (*i.e.*, $s-1$ times: contraction of any s indices with V_A gives zero by the Young properties of $\Omega_{(p)}{}^{A_0(s-1),\dots,A_q(s_q)}$) V-tangential component of $\Omega_{(p)}$. There is also a number of $s-2$ times V-tangential components of $\Omega_{(p)}$, which are called auxiliary fields. All other Lorentz-covariant components of field $\Omega_{(p)}$ are called extra fields.

Using the standard gauge for the compensator field $V^A=\delta^A{}_{d+1}$ [36] one can rewrite (2.8) with Lorentz indices $\lambda^{s-1}e_{(p)}{}^{a_1(s-1),\dots,a_q(s_q)} = V_{A_0}\cdots V_{A_0}\Omega_{(p)}{}^{A_0(s-1),a_1(s-1),\dots,a_q(s_q)}$. The frame-type field (2.8) is related to the metric-type field by the formula $\Phi^{a_1(s_1),a_2(s_2),\dots,a_q(s_q)}(x) = e_{(p)}^{a_1\dots a_p;\,a_1(s-1),\dots,a_p(s-1),a_{p+1}(s_{p+1}),\dots,a_q(s_q)}(x)$, where symmetrization over the indices denoted by the same letter is assumed, *i.e.* the metric-type field $\Phi(x)$ results from symmetrization of the p-form indices, converted into tangent ones, with tangent indices of the first p rows of the p-form field $e_{(p)}$. The gauge transformations (2.3) and (2.6) are related by the analogous formula.

It was proposed in [1] to look for the action of a free mixed-symmetry bosonic field in the form

$$\mathcal{S}_2 = \int_{\mathcal{M}^d} U^{\cdots}(V)\epsilon^{\cdots}{}_{M_1\dots M_{d-2p-2}N} E_0^{M_1}\wedge\cdots\wedge E_0^{M_{d-2p-2}} V^N \wedge\; R^{\cdots}_{(p+1)} \wedge\; R^{\cdots}_{(p+1)}\,, \quad (2.9)$$

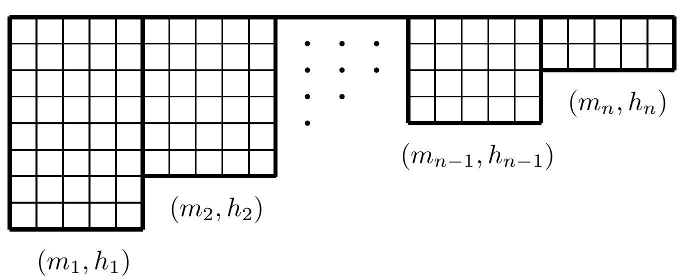
where $U^{\cdots}(V)$ are some coefficients which parameterize various types of index contractions between curvatures, compensators and the ϵ-tensor. Any such action is manifestly AdS covariant and gauge invariant with respect to the gauge transformations (2.6). In [1, 33] several examples of mixed-symmetry fields were described in terms of the actions of the form (2.9). In this letter we extend these results to massless bosonic fields of generic mixed-symmetry type.

2.2 Antisymmetric basis

Let us rewrite the gauge field $\Omega_{(p)}$ (2.4) in the antisymmetric basis as

$$\Omega_{(p)}^{A_1[\tilde{h}_1],\,\dots\,,A_{s-1}[\tilde{h}_{s-1}]}\,, \quad (2.10)$$

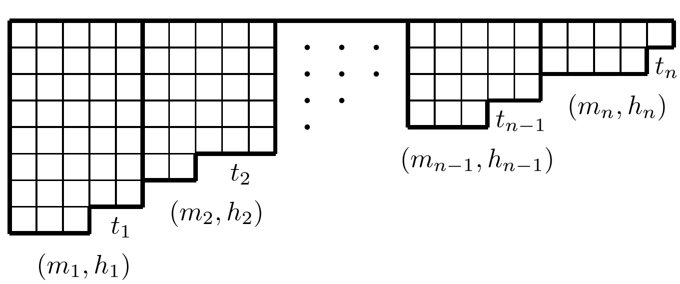
where $\tilde{h}_1 \geq \cdots \geq \tilde{h}_{s-1} \geq p+1$ are the heights of the columns of the $o(d-1,2)$ Young tableau corresponding to (2.4). Here $A_i[\tilde{h}_i]$ denotes antisymmetrized indices $A_i^1....A_i^{\tilde{h}_i}$ associated with the $\tilde{h}_i$-th column. In the antisymmetric basis it is required that antisymmetrization over all indices from a given group $A_i[\tilde{h}_i]$ with an index A_j gives zero once $j > i$. The symmetric and antisymmetric bases are equivalent being related by a linear map. Note that $\tilde{h}_1 \leq q = [(d-1)/2]$. It is convenient to combine columns of equal heights into vertical blocks. The result can be depicted as



(2.11)

Here I-th block has size (m_I,h_I), where m_I is its length and h_I is its height, $I = 1 \div n$. We have $q \geq h_1 = \tilde{h}_1 = \ldots = \tilde{h}_{m_1} > h_2 = \tilde{h}_{m_1+1} = \ldots = \tilde{h}_{m_1+m_2} > \ldots > h_n = \tilde{h}_{m_1+\ldots+m_{n-1}+1} = \ldots = \tilde{h}_{s-1} \geq p+1$.

Reduction of the traceless $o(d-1,2)$ Young tableau (2.11) with respect to the Lorentz subalgebra $o(d-1,1) \subset o(d-1,2)$ gives the set of traceless $o(d-1,1)$ Young tableaux of the following form



(2.12)

with various t_I such that $0 \le t_I \le m_I$, $I = 1 \div n$.

The set of Lorentz p-form gauge fields resulting from the AdS_d p-form field $\Omega_{(p)}$ consists of

- Physical field $e_{(p)}$ with $t_I = m_I$, $I = 1 \div n$ which corresponds to the Lorentz Young tableau (2.12) with the minimal number of cells.
- Auxiliary fields with $t_J = m_J - 1$ for some fixed J and $t_I = m_I$, $I \neq J$ which have one more cell compared to the Young tableau of the physical field. We distinguish between "relevant" auxiliary field $\omega_{(p)}$ with $t_1 = m_1 - 1$ and "irrelevant" auxiliary fields $\omega'_{(p)}$ with $t_J = m_J - 1$, $J \neq 1$.
- Extra fields $w_{(p)}$ with $\sum_{I=1}^{n} t_I \le s - 3$ which have two or more additional cells compared to the Young tableau of the physical field.

The characteristic feature of the relevant auxiliary field $\omega_{(p)}$ is that it is the most antisymmetric among all auxiliary fields, *i.e.* its first column is of maximal possible height h_1. This terminology for auxiliary fields is introduced to emphasize that they play different dynamical roles as will be explained in sect. 4.

In the antisymmetric basis, the formula (2.8) for the physical field $e_{(p)}$ is replaced by

$$\lambda^{s-1} e_{(p)}{}^{A_1[\tilde{h}_1-1],\dots,A_{s-1}[\tilde{h}_{s-1}-1]} = V_{A_1} \dots V_{A_{s-1}} \Omega_{(p)}{}^{A_1[\tilde{h}_1],\dots,A_{s-1}[\tilde{h}_{s-1}]} \,. \tag{2.13}$$

The expression for the relevant auxiliary field is

$$\begin{aligned}\lambda^{s-2} \omega_{(p)}^{A_1[\tilde{h}_1],A_2[\tilde{h}_2-1],\dots,A_{s-1}[\tilde{h}_{s-1}-1]} &= V_{A_2} \cdots V_{A_{s-1}} \Omega_{(p)}{}^{A_1[\tilde{h}_1],A_2[\tilde{h}_2],\dots,A_{s-1}[\tilde{h}_{s-1}]} \\ &\quad - \tilde{h}_1 V^{A_1} V_{A_2} \dots V_{A_{s-1}} V_B\, \Omega_{(p)}{}^{BA_1[\tilde{h}_1-1],\dots,A_{s-1}[\tilde{h}_{s-1}]} \,.\end{aligned} \tag{2.14}$$

These formulae for the physical and the relevant auxiliary fields contain no explicit Young symmetry projectors that makes the antisymmetric basis of the Young tableaux most convenient for our purposes. Note that in the case of gravity, the formula (2.14) reduces to $\omega^{AB} = \Omega^{AB} - \lambda\,(E^A\, V^B - E^B\, V^A)$ (see the beginning of sect. 2).

3 Generating Fock oscillator approach

Let us introduce the set of fermionic oscillators $\psi_\alpha{}^A = (\psi_i{}^A, \psi^{j\,A})$ and $\bar{\psi}_\alpha{}^A = (\bar{\psi}_i{}^A, \bar{\psi}^{j\,A})$ $(i, j = 1 \div (s-1),\ \alpha = 1 \div 2(s-1))$, which satisfy the anticommutation relations

$$\{\psi_i{}^A, \bar{\psi}^{jB}\} = \delta_i^j\, \eta^{AB}\,, \qquad\qquad \{\psi^{iA}, \bar{\psi}_j{}^B\} = \delta_j^i\, \eta^{AB} \tag{3.1}$$

with all other anticommutators equal to zero. Also introduce fermionic oscillators θ^A and $\bar{\theta}^B$ which satisfy anticommutation relations

$$\{\theta^A, \bar{\theta}^B\} = \eta^{AB}\,, \quad \{\theta^A, \theta^B\} = 0\,, \quad \{\bar{\theta}^A, \bar{\theta}^B\} = 0 \tag{3.2}$$

and anticommute with ${\psi_\alpha}^A$ and ${\bar\psi_\alpha}^A$.

Let us define the left and right Fock vacua by $\bar\psi^A_\alpha|0\rangle=0$, $\bar\theta^A|0\rangle=0$ and $\langle 0|\psi^A_\alpha=0$, $\langle 0|\bar\theta^A=0$ along with

$$\langle 0|\,\theta^{A_1}\cdots\theta^{A_{d+1}}|0\rangle=\epsilon^{A_1\cdots A_{d+1}}\,,\qquad \langle 0|\,\theta^{A_1}\cdots\theta^{A_k}|0\rangle=0\,,\ \ \text{for}\ \ k\neq d+1\,. \tag{3.3}$$

The oscillators θ provide a convenient way to introduce $o(d-1,2)$ ϵ-tensor via formula (3.3).

In our construction, p-form $o(d-1,2)$ gauge fields will be described as vectors of two types $|\hat\Omega_{(p)}\rangle=\hat\Omega_{(p)}|0\rangle$ and $|\breve\Omega_{(p)}\rangle=\breve\Omega_{(p)}|0\rangle$, where

$$\begin{aligned}
\hat\Omega_{(p)}&={\Omega_{(p)}}^{A_1[\tilde h_1],\dots,A_{s-1}[\tilde h_{s-1}]}(\psi^1_{A_1})^{\tilde h_1}\cdots(\psi^{s-1}_{A_{s-1}})^{\tilde h_{s-1}}\,,\\
\breve\Omega_{(p)}&=\Omega_{(p)\,A_1[\tilde h_1],\dots,A_{s-1}[\tilde h_{s-1}]}(\psi^{A_1}_1)^{\tilde h_1}\cdots(\psi^{A_{s-1}}_{s-1})^{\tilde h_{s-1}}\,.
\end{aligned}\tag{3.4}$$

More generally, operators $\hat A_{(m)}$ and $\breve A_{(m)}$ will be assumed to be analogously constructed from a m-form $A_{(m)}$ instead of $\Omega_{(p)}$. The Young symmetry and tracelessness conditions on the p-form gauge fields can be written as

$${l^i}_j|\hat\Omega_{(p)}\rangle=0\,,\quad i<j\,,\qquad\qquad \bar s_{ij}|\hat\Omega_{(p)}\rangle=0\,, \tag{3.5}$$

$${l_i}^j|\breve\Omega_{(p)}\rangle=0\,,\quad i<j\,,\qquad\qquad \bar s^{ij}|\breve\Omega_{(p)}\rangle=0\,, \tag{3.6}$$

$${l^i}_i|\hat\Omega_{(p)}\rangle=\tilde h_i|\hat\Omega_{(p)}\rangle,\qquad\qquad {l_i}^i|\breve\Omega_{(p)}\rangle=\tilde h_i|\breve\Omega_{(p)}\rangle \tag{3.7}$$

with

$$l_{\alpha\beta}=\eta_{AB}\,\psi^A_\alpha\bar\psi^B_\beta\,,\quad \bar s_{\alpha\beta}=\eta_{AB}\,\bar\psi^A_\alpha\bar\psi^B_\beta\,. \tag{3.8}$$

The linearized curvatures (2.5) are

$$|\hat R_{(p+1)}\rangle=\hat R_{(p+1)}|0\rangle=\mathsf{D}_0|\hat\Omega_{(p)}\rangle\,,\qquad |\breve R_{(p+1)}\rangle=\breve R_{(p+1)}|0\rangle=\mathsf{D}_0|\breve\Omega_{(p)}\rangle\,. \tag{3.9}$$

Here the $o(d-1,2)$ covariant background derivative is given by

$$\mathsf{D}_0=\mathrm{d}+{\Omega_0}^A{}_B\psi^i_A\bar\psi^B_i+{\Omega_0}_A{}^B\psi^A_i\bar\psi^i_B+{\Omega_0}^A{}_B\theta_A\bar\theta^B\,,\qquad \mathsf{D}_0^2=0\,, \tag{3.10}$$

where ${\Omega_0}^{AB}$ is the background AdS_d gauge field satisfying the zero-curvature condition (2.1). The gauge transformations (2.6) and Bianchi identities (2.7) are

$$\begin{aligned}
\delta|\hat\Omega_{(p)}\rangle&=\mathsf{D}_0|\hat\xi_{(p-1)}\rangle\,,\qquad\qquad &\mathsf{D}_0|\hat R_{(p+1)}\rangle=0\,,\\
\delta|\breve\Omega_{(p)}\rangle&=\mathsf{D}_0|\breve\xi_{(p-1)}\rangle\,,\qquad\qquad &\mathsf{D}_0|\breve R_{(p+1)}\rangle=0\,.
\end{aligned}\tag{3.11}$$

In the sequel we make use of the following operators

$$\bar\eta_\alpha=\bar\psi^A_\alpha\theta_A\,,\qquad \bar v_\alpha=\bar\psi^A_\alpha V_A\,,\qquad \chi=\theta^A V_A\,,\qquad \mathsf{E}_0=E^A_0\theta_A\,, \tag{3.12}$$

where V^A and E^A_0 are the compensator and the background frame field, respectively.

4 The higher-spin action

In the antisymmetric basis, the action functional $\mathcal{S}_2$ still has the form (2.9). With the help of the Fock notations of the previous section, it reads as

$$\mathcal{S}_2 = \int_{\mathcal{M}^d} \langle 0|(\wedge \mathsf{E}_0)^{d-2p-2}\chi\,\mathcal{U}(\bar{s},\bar{\eta},\bar{v}) \wedge \hat{R}_{(p+1)} \wedge \breve{R}_{(p+1)}|0\rangle\,, \tag{4.1}$$

where $\mathcal{U}(\bar{s},\bar{\eta},\bar{v})$ is some polynomial of $\bar{s}_{\alpha\beta}$, $\bar{\eta}_\alpha$ and $\bar{v}_\alpha$. The function $\mathcal{U}$ contains $2p+2$ oscillators θ which add up to $d-2p-1$ oscillators θ in $(\mathsf{E}_0)^{d-2p-2}\chi$ and generate the ϵ-tensor by (3.3). The operators E_0, χ and $\bar{\eta}$ realize contractions of the ϵ-tensor with the background frame field, compensator and HS curvatures, respectively. The operator $\bar{v}$ realizes contractions between the compensator and HS curvatures. The operator $\bar{s}$ realizes contractions between two HS curvatures. Using the symmetry of (4.1) with respect to exchange of the $(p+1)$-form HS curvatures we require $\mathcal{U}(\bar{s},\bar{\eta},\bar{v}) = \mathcal{U}(\bar{s},\bar{\eta}_i,\bar{\eta}^i,\bar{v}_i,\bar{v}^i)$ to satisfy the symmetry property

$$\mathcal{U}(\bar{s},\bar{\eta}_i,\bar{\eta}^i,\bar{v}_i,\bar{v}^i) = (-1)^{p+N+1}\mathcal{U}(-\bar{s},\bar{\eta}^i,\bar{\eta}_i,\bar{v}^i,\bar{v}_i)\,, \tag{4.2}$$

where N is defined by $\hat{\Omega}_{(p)}(\psi) = (-1)^N\hat{\Omega}_{(p)}(-\psi)$.

Taking into account (3.9) and (3.11), one can obtain (more details will be given in [37])

$$\begin{aligned}\delta\mathcal{S}_2 &= 2(-1)^{d+p}\int_{\mathcal{M}^d}\langle 0|\mathsf{D}_0\Big((\wedge \mathsf{E}_0)^{d-2p-2}\chi\,\mathcal{U}\Big) \wedge \hat{R}_{(p+1)} \wedge \delta\breve{\Omega}_{(p)}|0\rangle = \\ &= 2(-1)^p\frac{\lambda}{d-2p-1}\int_{\mathcal{M}^d}\langle 0|(\wedge \mathsf{E}_0)^{d-2p-1}\chi\,\mathcal{Q}\mathcal{U} \wedge \hat{R}_{(p+1)} \wedge \delta\breve{\Omega}_{(p)}|0\rangle\,,\end{aligned} \tag{4.3}$$

where

$$\mathcal{Q} = \Big(d-1+\bar{v}^\alpha\frac{\partial}{\partial\bar{v}^\alpha} - \bar{\eta}^\alpha\frac{\partial}{\partial\bar{\eta}^\alpha}\Big)\bar{v}^\beta\frac{\partial}{\partial\bar{\eta}^\beta} + \bar{s}^{\alpha\beta}\frac{\partial^2}{\partial\bar{v}^\alpha\partial\bar{\eta}^\beta}\,. \tag{4.4}$$

The important fact is that

$$\mathcal{Q}^2 = 0\,, \tag{4.5}$$

which is a consequence of $\mathsf{D}_0^2 = 0$. A natural guess is that, in an appropriate representation, $\mathcal{Q}$ can be rewritten as a de Rham operator. Indeed, one can see that

$$\delta = \bar{v}^\alpha\frac{\partial}{\partial\bar{\eta}^\alpha} = A^{-1}\mathcal{Q}A\,, \tag{4.6}$$

where

$$A = (d-1+\bar{v}^\alpha\frac{\partial}{\partial\bar{v}^\alpha} - \bar{\eta}^\alpha\frac{\partial}{\partial\bar{\eta}^\alpha})!!\exp(\frac{1}{2}\bar{s}^{\alpha\beta}\frac{\partial^2}{\partial\bar{v}^\alpha\partial\bar{v}^\beta})\,. \tag{4.7}$$

Since the variation of the action (4.1) has the form (4.3), total derivative terms in the action result from a $\mathcal{Q}$-closed function $\mathcal{U}$ that by virtue of the Poincare Lemma can be represented as

$$\mathcal{U}(\bar{s},\bar{\eta},\bar{v}) = \mathcal{Q}\mathcal{T}(\bar{s},\bar{\eta},\bar{v}) \tag{4.8}$$

for some $\mathcal{T}(\bar{s},\bar{\eta},\bar{v})$.

To find $\mathcal{U}$ up to $\mathcal{Q}$-closed terms we impose the extra field decoupling condition requiring that the variation of the action with respect to extra fields $w_{(p)}$ must vanish identically. This is equivalent to the condition that the resulting action does not contain higher derivatives. However, it turns out that this condition alone does not fix the action uniquely. The point is that it is not enough to require the AdS action to be invariant under AdS gauge symmetries and to contain first order derivatives in order to guarantee that it describes a correct number of degrees of freedom. Correct choice is dictated by the structure of the kinetic terms in the action which, for the metric-type field $\Phi(x)$, is given by $\mathcal{S}_2^{flat} \sim \int \partial\Phi\partial\Phi$ resulting from the AdS_d action $\mathcal{S}_2^{AdS} \sim \int \mathcal{D}\Phi\mathcal{D}\Phi + \lambda^2\Phi^2$ in the flat limit $\lambda \to 0$. In the AdS_d space, mass-like terms $\lambda^2\Phi^2$ break down all the gauge symmetries (2.2) of the action $\mathcal{S}_2^{flat}$ except for $\delta\Phi = \mathcal{D}S_p$ associated with the AdS_d gauge parameter S_p (2.3). Thus an additional condition is that the correct action must acquire all flat gauge symmetries in the flat limit. As we show this is achieved if the action is independent of the irrelevant auxiliary fields (more details will be given in [37]). So we impose the decoupling condition requiring the variation with respect to extra fields $w_{(p)}$ and irrelevant auxiliary fields $\omega'_{(p)}$ to be identically zero

$$\frac{\delta\mathcal{S}_2}{\delta w_{(p)}} \equiv 0\,, \qquad \frac{\delta\mathcal{S}_2}{\delta\omega'_{(p)}} \equiv 0\,. \tag{4.9}$$

In other words the condition (4.9) implies that the action depends non-trivially on the physical field and the relevant auxiliary field only. As we show in the rest of this letter the decoupling condition fixes a form of the action up to a normalization factor. Note that for symmetric HS fields [10] and for particular examples of mixed-symmetry fields considered in [1, 33] irrelevant auxiliary fields are absent so that the decoupling condition is equivalent to the extra field decoupling condition [10, 11].

To find the function $\mathcal{U}$ we adhere the following strategy. Firstly, we find the equations of motion in the form consistent with the decoupling condition. Secondly, we reconstruct the action function $\mathcal{U}$ that leads to these equations of motion.

For the subsequent analysis it is convenient to introduce a notion of weak equality. Two polynomials $\mathcal{A}(\bar{s},\bar{\eta},\bar{v})$ and $\mathcal{B}(\bar{s},\bar{\eta},\bar{v})$ are weakly equivalent $\mathcal{A}\sim\mathcal{B}$, if

$$\langle 0|(\wedge \mathsf{E}_0)^{d-m-n}\chi\Big(\mathcal{A}-\mathcal{B}\Big)\wedge \hat{A}_{(m)}\wedge \breve{B}_{(n)}|0\rangle = 0\,, \tag{4.10}$$

for any fields $A_{(m)}$ and $B_{(n)}$, which satisfy the Young symmetry and tracelessness properties (3.5)-(3.7). The meaning of the weak equivalence of two functions is that they coincide modulo terms proportional to Young symmetrizers and trace operators which are zero by (3.5)-(3.7). A generic weakly zero function has the form

$$\mathcal{W} = \sum_{i,j=1}^{s-1}\mathcal{W}^{ij}\bar{s}^{ij} + \sum_{i,j=1}^{s-1}\mathcal{W}_{ij}\bar{s}_{ij} + \sum_{i,j=1,\,j>i}^{s-1}[\mathcal{W}_i{}^j, l_i{}^j] + \sum_{i,j=1,\,j>i}^{s-1}[\mathcal{W}^i{}_j, l^i{}_j] +$$
$$+\sum_{i=1}^{s-1}\Big([\mathcal{W}_i, l_i{}^i] - \tilde{h}_i\mathcal{W}_i\Big) + \sum_{i=1}^{s-1}\Big([\mathcal{W}^i, l^i{}_i] - \tilde{h}_i\mathcal{W}^i\Big)\,, \tag{4.11}$$

where $\mathcal{W}_{ij}$, $\mathcal{W}^{ij}$, $\mathcal{W}_i{}^j$, $\mathcal{W}^i{}_j$, $\mathcal{W}_i$ and $\mathcal{W}^i$ are arbitrary functions of $\bar{s}$, $\bar{\eta}$ and $\bar{v}$. Here the first two terms are weakly zero due to the tracelessness condition (3.5) and the other terms are weakly zero due to the Young symmetry properties (3.6), (3.7). The important property is that the operators $\bar{s}_{ij}$, $\bar{s}^{ij}$, $l_i{}^j$ and $l^i{}_j$ commute with $\mathcal{Q}$. As a result, $\mathcal{Q}\mathcal{W} \sim 0$, $\forall\, \mathcal{W} \sim 0$.

5 Equations of motion

Let the variation (4.3) have the form

$$\delta \mathcal{S}_2 = \int_{\mathcal{M}^d} \langle 0|(\wedge \mathsf{E}_0)^{d-2p-1} \chi \mathcal{E}(\bar{s}, \bar{\eta}, \bar{v}) \wedge \hat{R}_{(p+1)} \wedge \delta \breve{\Omega}_{(p)}|0\rangle \,, \tag{5.1}$$

for some polynomial $\mathcal{E}(\bar{s}, \bar{\eta}, \bar{v})$. For the function $\mathcal{E}$ to result from the variation of some action (4.1), it must be weakly $\mathcal{Q}$-exact

$$\mathcal{E} \sim \mathcal{Q}\mathcal{U} \,. \tag{5.2}$$

Since $\mathcal{Q}^2 = 0$ and $\mathcal{Q}$ maps weakly zero functions to weakly zero functions, $\mathcal{E}$ has to satisfy consistency condition

$$\mathcal{Q}\mathcal{E} \sim 0 \,. \tag{5.3}$$

Now our goal is to find $\mathcal{E}$ satisfying the decoupling condition (4.9) and the consistency condition (5.3) and then to reconstruct $\mathcal{U}$ via (5.2).

A function $\mathcal{E}$ satisfying the decoupling condition has the form

$$\mathcal{E}(\bar{s}, \bar{\eta}, \bar{v}) = \Big(\bar{\eta}_1 \frac{\partial}{\partial \bar{v}_1} - \bar{\eta}^1 \frac{\partial}{\partial \bar{v}^1}\Big) \tilde{\mathcal{E}}(\bar{s}, \bar{\eta}) \bar{v}^{2(s-1)} \,, \tag{5.4}$$

where $\bar{v}^{2(s-1)} = \bar{v}_1 \cdots \bar{v}_{s-1} \bar{v}^1 \cdots \bar{v}^{s-1}$. Actually, the first term in (5.4) contains $(s-1)$ compensators (which is a maximal possible number) contracted with $\delta\Omega_{(p)}$ and therefore corresponds to the variation with respect to the physical field (cf. (2.13)). Analogously, the second term contains $(s-2)$ compensators contracted with all columns of $\delta\Omega_{(p)}$ except for the first one and therefore corresponds to the variation with respect to the relevant auxiliary field (cf. (2.14)). In the both cases, the remaining index in the first column of either $R_{(p+1)}$ or $\delta\Omega_{(p)}$ is contracted with the ϵ-tensor by $\bar{\eta}_1$ or $\bar{\eta}^1$. The relative coefficient in (5.4) is fixed by the symmetry property of $\mathcal{U}$ (4.2).

The important fact is that, using the Young symmetry properties of the gauge fields, *i.e.* by adding weakly zero terms, the function $\tilde{\mathcal{E}}$ can always be chosen in the form

$$\tilde{\mathcal{E}}(\bar{u}, \bar{n}) = \Big(\prod_{i=1}^{s-1} (\bar{u}_i)^{\tilde{h}_i - 1}\Big) \sum_{\substack{p_I \geq 0,\, I=1\div n \\ p_1+\cdots+p_n = p}} \rho(p_1, \ldots, p_n) \Big(\frac{\bar{n}_{\mu_1}}{\bar{u}_{\mu_1}}\Big)^{p_1} \cdots \Big(\frac{\bar{n}_n}{\bar{u}_{\mu_n}}\Big)^{p_n} \,, \tag{5.5}$$

where μ_I is a number of the first column of the I-the block (*i.e.* $\mu_1 = 1$, $\mu_2 = m_1 + 1$,..., $\mu_n = m_1 + \cdots + m_{n-1} + 1$) and new variables

$$\bar{u}_i = \bar{s}_i{}^i \,, \qquad \bar{n}_I = \bar{\eta}_{\mu_I} \bar{\eta}^{\mu_I} \,, \tag{5.6}$$

(no sums over repeated indices) realize column-to-column contractions between $R_{(p+1)}$ and $\delta\Omega_{(p)}$ and contractions of the ϵ-tensor with the first columns of I-th vertical blocks of $R_{(p+1)}$ and $\delta\Omega_{(p)}$. The coefficients $\rho(p_1, \ldots, p_n)$ parameterize various types of contractions between $2p$ indices of the ϵ-tensor and those of $R_{(p+1)}$, $\delta\Omega_{(p)}$.

Let us now solve the equation (5.3). Leaving details for [37] we give the final result of the substitution of the ansatz (5.4) into the equation (5.3). Modulo weakly zero terms we obtain the following system of equations

$$\begin{aligned}
&\Bigg(\frac{(\bar{N}_1 + 2)(\bar{N}_I + m_I)}{\bar{U}_1(\bar{N}_I + 1)} \bar{u}_{\mu_1} \frac{\partial}{\partial \bar{n}_1} + \sum_{J=2}^{I-1} \frac{(\bar{N}_J + 1)(\bar{N}_I + m_I)}{\bar{U}_J(\bar{N}_I + 1)} \bar{u}_{\mu_J} \frac{\partial}{\partial \bar{n}_J} + \\
&+ \Big(\sum_{J=I+1}^{n} \frac{\bar{N}_J}{\bar{U}_I} - \frac{s - \mu_I - m_I}{\bar{U}_I} - 1 \Big) \bar{u}_{\mu_I} \frac{\partial}{\partial \bar{n}_I} \Bigg) \tilde{\mathcal{E}}(\bar{u}, \bar{n}) = 0 \,, \qquad 2 \leq I \leq n \,,
\end{aligned} \tag{5.7}$$

where

$$\bar{N}_I = \bar{n}_I \frac{\partial}{\partial \bar{n}_I} \,, \qquad \bar{U}_I = \bar{u}_{\mu_I} \frac{\partial}{\partial \bar{u}_{\mu_I}} \,, \quad \text{no summation} \,. \tag{5.8}$$

Taking into account (5.5), eq. (5.7) can be reduced to a system of recurrent equations

$$\rho(p_1 - 1, ..., p_I + 1, ..., p_n) = G_I(p_1, \; ..., p_I, ..., p_n) \, \rho(p_1, ..., p_I, ..., p_n) \,, \quad I = 2 \div n \,, \tag{5.9}$$

$$p_1 \geq 1 \,, \qquad p_I \geq 0 \,, \quad I = 2 \div n \,, \qquad p_1 + ... + p_n = p \,, \tag{5.10}$$

where

$$G_I(p_1, ..., p_n) = \frac{(p_I - h_I + 1)(p_I + m_I)}{(p_I + 1)^2} \frac{p_1(p_1 + 1)}{h_1 - p_1} \frac{\prod_{J=2}^{I-1} \Big(\sum_{K=J+1}^{n} p_K - \vartheta(J) - m_J \Big)}{\prod_{J=2}^{I} \Big(\sum_{K=J}^{n} p_K - \vartheta(J) \Big)} \tag{5.11}$$

and $\vartheta(I) = s - \mu_I - m_I + h_I - 1$. Note that the arguments of the factorials in the numerators of (5.11) are strictly non-negative in the range (5.10). The general solution is

$$\begin{aligned}
&\rho(p_1, ..., p_n) = \\
&= \frac{\rho \, \delta(p - \sum_{I=1}^{n} p_I)}{(p_1!)^2 (p_1 + 1)(h_1 - p_1 - 1)!} \prod_{I=2}^{n} \frac{(p_I + m_I - 1)!}{(p_I!)^2 \, (h_I - p_I - 1)!} \frac{\Big(\vartheta(I) - \sum_{J=I}^{n} p_J \Big)!}{\Big(\vartheta(I) + m_I - \sum_{J=I+1}^{n} p_J \Big)!} \,,
\end{aligned} \tag{5.12}$$

where ρ is an arbitrary constant. It is elementary to check that the function (5.12) does solve the system (5.9)-(5.11).

Expressions (5.4), (5.5), (5.12) determine function $\mathcal{E}$ satisfying the weak $\mathcal{Q}$-closedness equation (5.3) and give rise to the variation (5.1) satisfying the decoupling condition (4.9). The constructed $\mathcal{E}(\bar{s}, \bar{\eta}, \bar{v})$ is (weakly) unique up to a normalization factor ρ.

6 Reconstruction of the action

Having found the function $\mathcal{E}$ we now solve the eq. (5.2) on the action function $\mathcal{U}$. To this end, we rewrite the $\mathcal{Q}$-closedness condition (5.3) as a strong equality

$$\mathcal{Q}\mathcal{E} = \mathcal{P}\,, \tag{6.1}$$

where $\mathcal{P}$ represents some weakly zero terms

$$\mathcal{P} \sim 0\,. \tag{6.2}$$

From (6.1) we have that $\mathcal{Q}\mathcal{P} = 0$. As the operator $\mathcal{Q}$ (4.6) is equivalent to the de Rham operator δ, by Poincare Lemma the function $\mathcal{P}$ is $\mathcal{Q}$-exact

$$\mathcal{P} = \mathcal{Q}\mathcal{K}\,. \tag{6.3}$$

Let us show that $\mathcal{K}$ in (6.3) can be chosen to be weakly zero. Indeed, taking into account (4.6), we rewrite eq. (6.3) as

$$\mathcal{P}' = \delta\mathcal{K}'\,, \tag{6.4}$$

where $\mathcal{P}' = A^{-1}\mathcal{P}$, $\mathcal{K}' = A^{-1}\mathcal{K}$ and the operator A is given by (4.7). Consider the operator δ^*

$$\delta^* = \bar{\eta}^\alpha \frac{\partial}{\partial \bar{v}^\alpha}\,, \qquad \delta^{*\,2} = 0\,. \tag{6.5}$$

One obtains that

$$\Delta \equiv \{\delta, \delta^*\} = \bar{\eta}^\alpha \frac{\partial}{\partial \bar{\eta}^\alpha} + \bar{v}^\alpha \frac{\partial}{\partial \bar{v}^\alpha}\,. \tag{6.6}$$

Acting by δ^* at the both sides of (6.4) one obtains

$$\Delta\mathcal{K}' = \delta^*\mathcal{P}' + \delta\delta^*\mathcal{K}'\,. \tag{6.7}$$

The operator Δ commutes with δ and δ^*. As a result, a partial solution of (6.4) is

$$\mathcal{K}' = \Delta^{-1}\delta^*\mathcal{P}'\,. \tag{6.8}$$

The operator Δ^{-1} admits the following integral realization

$$\Delta^{-1}\mathcal{A}(\bar{s}, \bar{\eta}, \bar{v}) = \int_0^1 \frac{\mathrm{d}t}{t}\mathcal{A}(\bar{s}, t\bar{\eta}, t\bar{v}) \tag{6.9}$$

for a function $\mathcal{A}(\bar{s},\bar{\eta},\bar{v})$ such that $t^{-1}A(\bar{s},t\bar{\eta},t\bar{v})$ is polynomial in t. Substituting (6.9) into (6.8) one obtains

$$\mathcal{K}'(\bar{s},\bar{\eta},\bar{v}) = \int_0^1 \frac{\mathrm{d}t}{t}\bar{\eta}^\beta \frac{\partial}{\partial \bar{v}^\beta}\mathcal{P}'(\bar{s},t\bar{\eta},t\bar{v})\,. \tag{6.10}$$

Equivalently,

$$\mathcal{K}(\bar{s},\bar{\eta},\bar{v}) = A\Big\{\int_0^1 \frac{\mathrm{d}t}{t}\bar{\eta}^\beta \frac{\partial}{\partial \bar{v}^\beta}\Big(A^{-1}\mathcal{P}\Big)(\bar{s},t\bar{\eta},t\bar{v})\Big\}\,. \tag{6.11}$$

As the operators δ^*, A, A^{-1}, Δ, Δ^{-1} commute with the operators $\bar{s}_{ij}$, $\bar{s}^{ij}$, $l_i{}^j$, $l^i{}_j$ and $\mathcal{P}$ is weakly zero (6.2), we conclude from (4.11) that $\mathcal{K}$ of the form (6.11) is weakly zero as well.

Now we are in a position to solve eq. (5.2). Rewriting eq. (5.2) as a strong equality

$$\mathcal{Q}\mathcal{U} = \mathcal{E} - \mathcal{K}\,, \tag{6.12}$$

we find analogously to (6.4) that

$$\mathcal{U}(\bar{s},\bar{\eta},\bar{v}) = A\Big\{\int_0^1 \frac{\mathrm{d}t}{t}\bar{\eta}^\beta \frac{\partial}{\partial \bar{v}^\beta}\Big(\Big(A^{-1}\mathcal{E}\Big)(\bar{s},t\bar{\eta},t\bar{v}) - \Big(A^{-1}\mathcal{K}\Big)(\bar{s},t\bar{\eta},t\bar{v})\Big)\Big\}\,. \tag{6.13}$$

Substituting (6.13) into the action (4.1) one finally obtains

$$\mathcal{S}_2 = \lambda^{-2(s-1)}\int_{\mathcal{M}^d}\langle 0|(\wedge \mathsf{E}_0)^{d-2p-2}\chi A\Big\{\int_0^1 \frac{\mathrm{d}t}{t}\bar{\eta}^\beta \frac{\partial}{\partial \bar{v}^\beta}\Big(A^{-1}\mathcal{E}\Big)(\bar{s},t\bar{\eta},t\bar{v})\Big\}\wedge\hat{R}_{(p+1)}\wedge\breve{R}_{(p+1)}|0\rangle\,, \tag{6.14}$$

where the weakly zero term with $\mathcal{K}$ does not contribute to the action and the factor of $\lambda^{-2(s-1)}$ is introduced to provide correct flat limit. The action (6.14) satisfies the decoupling condition and is defined uniquely modulo total derivatives (4.8).

7 Flat space gauge symmetries

As explained in sect. 4 the constructed AdS_d HS action (6.14) should describe correct dynamics of a mixed-symmetry gauge field in the flat limit $\lambda = 0$ and hence has to be invariant under additional set of gauge symmetries (2.2). Here we show that in the flat limit of the AdS_d action (6.14) a gauge symmetry enhancement indeed takes place with respect to traceless flat space gauge parameters.

Using the standard gauge $V^A = \delta^A{}_{d+1}$ [36], setting the frame field to $E^a_{0\underline{n}} = \delta^a_{\underline{n}}$, Lorentz spin connection to $\omega^{ab}_{\underline{n}} = 0$ and replacing the Lorentz covariant derivative $\mathcal{D}$ with the flat derivative ∂, one rewrites the variation (5.1) in the form

$$\delta\mathcal{S}_2^{\mathrm{flat}} = \int_{\mathcal{M}^d}\Big(\langle 0|(\wedge \mathsf{E}_0)^{d-2p-1}\,\chi\tilde{\mathcal{E}}(\bar{u},\bar{\eta})\,\bar{\eta}^1\,\wedge\hat{r}_{(p+1)}\wedge\delta\breve{\omega}_{(p)}|0\rangle - \\ -\langle 0|(\wedge \mathsf{E}_0)^{d-2p-1}\,\chi\tilde{\mathcal{E}}(\bar{u},\bar{\eta})\,\bar{\eta}_1\,\wedge\hat{\mathcal{R}}_{(p+1)}\wedge\delta\breve{e}_{(p)}|0\rangle\Big)\,, \tag{7.1}$$

where the $(p+1)$-forms $r_{(p+1)} = \mathrm{d}e_{(p)} + \cdots$ and $\mathcal{R}_{(p+1)} = \mathrm{d}\omega_{(p)} + \cdots$ are the Lorentz components of the curvature $R_{(p+1)}$ associated with the physical and relevant auxiliary fields, respectively. The variation over the relevant auxiliary field $\omega_{(p)}$ gives rise to the equation of motion which can be cast into the following component form (for more details see [37])

$$r^{a_1[\tilde{h}_1-1],\ldots,a_{s-1}[\tilde{h}_{s-1}-1];\,m[p+1]}(x) = C^{a_1[\tilde{h}_1-1],\ldots,a_{s-1}[\tilde{h}_{s-1}-1],m[p+1]}(x)\,, \tag{7.2}$$

where world indices are converted into the tangent indices m. It was argued in Ref. [1] that the tensor $C^{a_1[\tilde{h}_1-1],\ldots,a_{s-1}[\tilde{h}_{s-1}-1],m[p+1]}(x)$ is either zero if $\tilde{h}_{s-1} = p+1$ or equals to the primary Weyl tensor if $\tilde{h}_{s-1} \neq p+1$. Equation (7.2) expresses the relevant auxiliary field $\omega_{(p)}$ in terms of first derivatives of the physical field $e_{(p)}$ up to pure gauge degrees of freedom. It is convenient to use the 1.5-formalism with the auxiliary field expressed implicitly by virtue of its equation of motion through the physical field.

The next step is to study how the frame-type physical field is transformed under the flat gauge symmetries. The flat gauge parameters $S_{(I)}, I = 1 \div n_\Phi$ (2.2) result from cutting a cell from I-th block of the metric-type field $\Phi(x)$. As a result, there is as many independent flat gauge parameters as the number of different vertical blocks of the Young tableau corresponding to the field $\Phi(x)$. Note that the gauge parameter $S_{(n_\Phi)}$ associated with the vertical block of the minimal height corresponds to the physical Lorentz-covariant component of the AdS_d gauge parameter $\xi_{(p-1)}$ (2.6).

It is convenient to convert all world indices originally carried by differential forms into tangent ones, contracting them in the operators $\hat{A}_{(n)}$=$(\delta\hat{\Omega}_{(p)},\, \hat{R}_{(p+1)},\, \hat{S}_{(I)})$ and $\breve{A}_{(n)}$=$(\delta\breve{\Omega}_{(p)},\, \breve{R}_{(p+1)},\, \breve{S}_{(I)})$ with the oscillators $\kappa^{\bullet}_a$, $\bar{\kappa}^a_{\bullet}$ and $\kappa^a_{\bullet}$, $\bar{\kappa}^{\bullet}_a$, which anticommute with the previously introduced oscillators and satisfy the anticommutation relations

$$\{\bar{\kappa}^a_{\bullet}, \kappa^{\bullet b}\} = \eta^{ab}\,, \qquad \{\bar{\kappa}^{\bullet a}, \kappa^b_{\bullet}\} = \eta^{ab}\,, \tag{7.3}$$

with other anticommutators being zero. The Fock vacua are defined by $\langle 0|\kappa^a_{\bullet} = 0$, $\langle 0|\kappa^{\bullet a} = 0$, $\bar{\kappa}^a_{\bullet}|0\rangle = 0$ and $\bar{\kappa}^{\bullet a}|0\rangle = 0$.

A flat gauge transformation of the metric-type field $\Phi(x)$ results as an appropriate projection of the following transformation of the physical field $e_{(p)}$

$$\delta_{(I)}\breve{e}_{(p)}|0\rangle = \mathcal{P} D^{\mu_I}\breve{S}_{(I)}|0\rangle\,, \tag{7.4}$$

where $D_{\mu_I} = \psi^a_{\mu_I}\partial_a$ and $\mathcal{P}$ projects r.h.s. of (7.4) on the Young tableau associated with tangent indices of the field $e_{(p)}$.

Substituting (7.4) into (7.1) and neglecting terms with the variation of the auxiliary field by using the 1.5-order formalism, one gets the variation with respect to the parameter $S_{(I)}$

$$\delta_{(I)}\mathcal{S}_2^{\text{flat}} = \int_{\mathcal{M}^d} \Big(\alpha_1\Delta_1 + \alpha_2\Delta_2 + \alpha_3\Delta_3\Big)\,, \tag{7.5}$$

where $\alpha_{1,2,3}$ are some coefficients and

$$\Delta_1 = \frac{1}{p}\langle 0|(\mathsf{E}_0)^d \chi \mathcal{F}_{(I)}(\bar{u})\bar{D}_1\bar{D}_I(\bar{K})^p\hat{\omega}_{(p)}\check{S}_{(I)}|0\rangle \,,$$

$$\Delta_2 = -\langle 0|(\mathsf{E}_0)^d \chi \mathcal{F}_{(I)}(\bar{u})\bar{D}_1\bar{M}_I\tilde{D}(\bar{K})^{p-1}\hat{\omega}_{(p)}\check{S}_{(I)}|0\rangle \,, \tag{7.6}$$

$$\Delta_3 = \langle 0|(\mathsf{E}_0)^d \chi \mathcal{F}_{(I)}(\bar{u})\bar{M}_1\bar{D}_I\tilde{D}(\bar{K})^{p-1}\hat{\omega}_{(p)}\check{S}_{(I)}|0\rangle$$

with

$$\bar{D}_I = \bar{\psi}_{\mu_I}{}^a\partial_a \,, \quad \tilde{D} = \bar{\kappa}^{\bullet}_a\partial^a \,, \quad \bar{M}_I = \bar{\kappa}^a_{\bullet}\bar{\psi}_{\mu_I a} \,, \quad \bar{K} = \bar{\kappa}^a_{\bullet}\bar{\kappa}^{\bullet}_a \,, \quad \mathcal{F}_{(I)}(\bar{u}) = \frac{1}{\bar{u}_{\mu_I}}\prod_{i=1}^{s-1}(\bar{u}_i)^{\tilde{h}_i - 1}. \tag{7.7}$$

Then one can show that the coefficients $\alpha_{1,2,3}$ satisfy the linear relation

$$\alpha_1 + \alpha_2 + \alpha_3 = 0 \tag{7.8}$$

resulting from the invariance of the original AdS_d action under Lorentz-type gauge symmetry acting on the frame-type HS field with the Lorentz gauge parameter $\xi_{(p-1)}$ of the same Young symmetry type as the relevant auxiliary field.

On the other hand, the operators $\Delta_{1,2,3}$ satisfy relations resulting from Bianchi identities. Indeed, consider the particular Bianchi identity at $\lambda = 0$

$$\mathrm{d}r_{(p+1)} + \sigma_- \mathcal{R}_{(p+1)} + ... = 0 \,. \tag{7.9}$$

Here σ_- is the operator decreasing a number of Lorentz indices and dots stand for contributions of the curvatures of the same Young symmetry as irrelevant auxiliary fields that can be disregarded in the flat limit by virtue of the decoupling condition (4.9). Taking into account the eq. (7.2) and projecting out the contribution of the Weyl tensor into Bianchi identity (7.9) one can prove that

$$\Delta_1 = \Delta_2 = \Delta_3 \tag{7.10}$$

(more details will be given in [37]). Comparing the relations (7.8) and (7.10) one concludes that the variation (7.5) is zero. Thus the constructed AdS_d action functional (6.14) exhibits in the flat limit additional gauge symmetries associated with the corresponding mixed-symmetry gauge field on Minkowski space.

8 Conclusions

In this letter we have announced the covariant Lagrangian formulation for a generic mixed-symmetry bosonic gauge field propagating on the AdS_d background and corresponding to a unitary $o(d-1,2)$-module. A novel feature of gauge fields of mixed-symmetry type is that, apart from the previously known extra field decoupling condition, it is necessary to require all but one auxiliary fields also not to contribute to

the action. The relevant auxiliary field contributing to the action is the maximally antisymmetric one. This condition fixes the Lagrangian modulo total derivative terms and guarantees correct flat gauge symmetry enhancement that, in its turn, implies correct counting of degrees of freedom.

The next step for the further study is to generalize the results of this letter to the case of fermionic fields. Also the unfolded formulation for mixed-symmetry fields is to be developed since it clarifies a structure of non-Abelian HS symmetries and of a non-linear HS theory with mixed-symmetry HS gauge fields. We plan to discuss these problems in the future publication [37].

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